Exercise	1	2	Total
100%	8	8	16
Points			

Extragalactic Astronomy and Cosmology

Homework 5 - Lecture 12 - Cosmic Dynamics

Due date: October 17

1 A Universe of stars

Suppose that the Universe were full of stars like the sun, each with $R = R_\odot = 6.96 \times 10^8\,\mathrm{m}$ and $m = M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$. If the stars were distributed uniformly throughout the Universe, what number density of stars would be required to make the density equal to the critical density (non-relativistic case) ? Given this density of stars, how far would you be able to see, on average, before your line of sight intersected a star? In fact, we can see galaxies at a distance $d \sim c/H_0 \sim 4000\,\mathrm{Mpc}$; does the transparency of the Universe on this length scale place useful limits on the number density of stars?

2 Wave-particle duality

The principle of wave-particle duality tells us that a particle with momentum p has an associated de Broglie wavelength of $\lambda = h/p$; this wavelength increases as $\lambda \propto a$, as the Universe expands. The total energy density of a gas of particles can be written as $\epsilon = n \cdot E$, where n is the number desnity of particles, and E is the energy per particle. For simplicity, let's assume that all the gas particles have the same mass m and momentum p. The energy per particle is then simply

$$E = \sqrt{m^2c^4 + p^2c^2} = \sqrt{m^2c^4 + h^2c^2/\lambda^2} \tag{1}$$

Compute the equation-of-state parameter w for this gas as a function of the scale factor a. Show that $w = \frac{1}{3}$ in the highly relativistic limit ($a \to 0$, $p \to \infty$) and that $w = 0$ in the highly nonrelativistic limit ($a \to \infty$, $p \to 0$).